On 1.4.2010, investment advisors indicate (a) that the dividends from M Ltd. and N Ltd. for the year ending 31.3.2011 are likely to be 20% and 35%, respectively and (b) that the probabilities of market quotations on 31.3.2011 are as below:

Probability factor	Price/share of M Ltd.	Price/share of N Ltd.
0.2	220	290
0.5	250	310
0.3	280	330

You are required to:

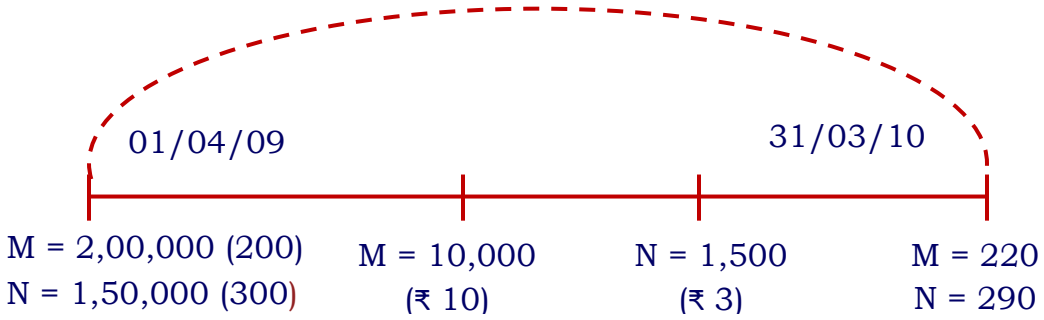
(i) Calculate the average return from the portfolio for the year ended 31.3.2010;

(ii) Calculate the expected average return from the portfolio for the year 2010-11; and

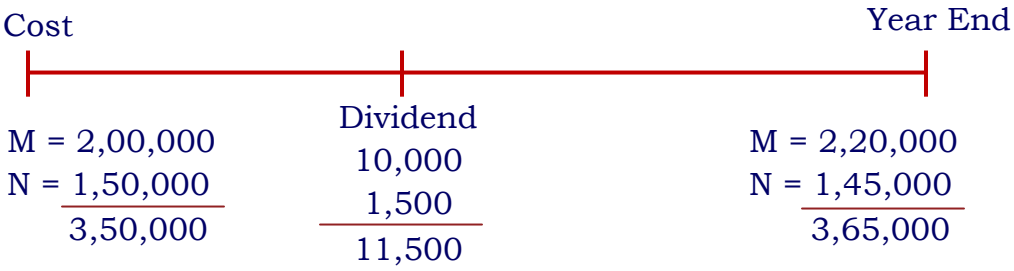
(iii) Advise X Co. Ltd., of the comparative risk in the two investments by calculating the standard deviation in each case.

(SM TYK – 32)

Solution:



(1) Portfolio Return [2009 – 10]



$$\text{Portfolio} = \frac{(3,65,000 - 3,50,000) + 11,500}{3,50,000} \times 100$$

$$= 7.57\%$$

Alternative

$$\text{Stock M} = \frac{(220 - 200) + 10}{200} \times 100 = 15\%$$

$$\text{Stock N} = \frac{(290 - 300) + 3}{300} \times 100 = -2.34\%$$

$$W_M = \frac{2,00,000}{3,50,000} = 0.571 \qquad W_N = \frac{1,50,000}{3,50,000} = 0.429$$

Portfolio Return = (15 × 0.571) + (-2.34 × 0.429) = 7.56%

(2) 2010 – 2011

M = 220
N = 290

M = ₹ 20
N = ₹ 3.50

M = (220 × 0.2) + (250 × 0.5) + (280 × 0.3) = 253

N = (290 × 0.2) + (310 × 0.5) + (330 × 0.3) = 312

$$ER_M = \frac{(253 - 220) + 20}{220} \times 100 = 24.10\%$$

$$ER_N = \frac{(312 - 290) + 3.50}{290} \times 100 = 8.79\%$$

$$W_M = \frac{2,20,000}{3,65,000} = 0.603$$

$$W_N = \frac{1,45,000}{3,65,000} = 0.397$$

$$ER_P = (24.10 \times 0.603) + (3.79 \times 0.397) = 18.02\%$$

(3) Standard Deviation

M Ltd $= \frac{(220-220)+20}{220} \times 100 = 9.09\%$

$= \frac{(250-220)+20}{220} \times 100 = 22.73\%$

$= \frac{(280-220)+20}{220} \times 100 = 36.36\%$

N Ltd $= \frac{(290-290)+3.5}{290} \times 100 = 1.25\%$

$= \frac{(310-290)+3.5}{220} \times 100 = 8.10\%$

$= \frac{(330-290)+3.5}{290} \times 100 = 15\%$

P	X	P (X)	$(X-\bar{X})$	$(X-\bar{X})^2$ P	Y	P (Y)	$(Y-\bar{Y})$	$(Y-\bar{Y})^2P$
0.2	9.09	1.82	-15.01	45.06	1.21	0.24	- 7.582	11.50
0.5	22.73	11.37	-1.37	0.94	8.10	4.04	- 0.692	0.24
0.3	36.36	10.91	12.26	45.09	15	4.5	6.208	11.56
	∑P (X) =	24.10		91.09		8.792		23.30

$\sigma x = \sqrt{91.09} = 9.54\%$

$\sigma y = \sqrt{23.30} = 4.83\%$

Standard Deviation of M Ltd. is more than Standard Deviation of N Ltd.

Hence, stock of M Ltd. is more risky.

18.50%	20.00%	Under Valued	Buy
13.00%	13.00%	Correctly Valued	Hold
16.00%	17.00%	Under Valued	Buy

EFFICIENT & OPTIMUM PORTFOLIO

Question – 09

Following is the data regarding six securities:

	A	B	C	D	E	F
Return (%)	8	8	12	4	9	8
Risk (Standard deviation)	4	5	12	4	5	6

(i) Assuming three will have to be selected, state which ones will be picked.

(ii) Assuming perfect correlation, show whether it is preferable to invest 75% in A and 25% in C or to invest 100% in E

(SM TYK – 05)

Solution:

(i) Efficient Securities

- Security B & F are inefficient because Security A provides same return at lower level of risk.
- Security D is inefficient because security A provides higher return at the same level of risk

Hence efficient stocks are A, C & E.

(ii) 75% in A & 25% in C

ER_p = (8 × 0.75) + (12 × 0.25) = 9%

SD_p = (4 × 0.75) + (12 + 0.25) = 6%

100 % in E

ER = 9%

S.D = 5%

Investment in 100% in stock E is better than portfolio because Stock E provides same return at lower level risk.

Question – 10

Suppose that economy A is growing rapidly and you are managing a global equity fund and so far you have invested only in developed-country stocks only. Now you have decided to add stocks of economy A to your portfolio. The table

below shows the expected rates of return, standard deviations, and correlation coefficients (all estimates are for aggregate stock market of developed countries and stock market of Economy A).

	Developed Country Stocks	Stocks of Economy A
Expected rate of return (annualized percentage)	10	15
Risk [Annualized Standard Deviation (%)]	16	30
Correlation Coefficient (ρ)	0.30	

Assuming the risk-free interest rate to be 3%, you are required to determine:

(a) What percentage of your portfolio should you allocate to stocks of Economy A if you want to increase the expected rate of return on your portfolio by 0.5%?

(b) What will be the standard deviation of your portfolio assuming that stocks of Economy A are included in the portfolio as calculated above?

(c) Also show how well the Fund will be compensated for the risk undertaken due to inclusion of stocks of Economy A in the portfolio?

(SM TYK – 24)

Solution:

(1) Calculation of weights

$ERP = ER_A \times W_A + ER_B \times W_B$

$10.5 = 10W_A + 15 (1 - W_A)$

$10.5 = 10 W_A + 15 - 15 W_A$

$W_A = \frac{4.5}{5} = 0.9$

$W_B = 0.1$

Developed Country = 90%

Economy A = 10%

(2) SD of Portfolio

$$\sigma_p = \sqrt{\sigma_A^2 W_A^2 + \sigma_B^2 W_B^2 + 2 \times \sigma_A \times W_A \times \sigma_B \times W_B \times r_{AB}}$$

$$= \sqrt{16^2 \times 0.9^2 + 30^2 \times 0.10^2 + 2 \times 16 \times 0.9 \times 30 \times 0.10 \times 0.3}$$

$$= 15.56\%$$

(3) Sharpe ratio

$$SR = \frac{ER - RF}{\sigma}$$

$$\text{Develop country only} = \frac{10 - 3}{16} = 0.4375$$

$$\text{Portfolio} = \frac{10.5 - 3}{15.56} = 0.482$$

Sharpe ratio will improve.

MINIMUM VARIANCE PORTFOLIO

Question – 11

An investor has two portfolios known to be on minimum variance set for a population of three securities A, B and C having below mentioned weights:

	WA	WB	WC
Portfolio X	0.30	0.40	0.30
Portfolio Y	0.20	0.50	0.30

It is supposed that there are no restrictions on short sales.

(i) What would be the weight for each stock for a portfolio constructed by investing ₹ 5,000 in portfolio X and ₹ 3,000 in portfolio Y?.

(ii) Suppose the investor invests ₹ 4,000 out of ₹ 8,000 in security A. How he will allocate the balance between security B and C to ensure that his portfolio is on minimum variance set?

(SM TYK – 31)

Solution:

(I) **PORTFOLIO (8,000)**

Portfolio X (5,000) **Portfolio Y (3,000)**

A	5,000 × 0.3	= 1,500	A	3,000 × 0.20	= 600
B	5,000 × 0.4	= 2,000	B	3,000 × 0.50	= 1,500
C	5,000 × 0.3	= 1,500	C	3,000 × 0.30	= 900

$$W_A = \frac{1,500 + 600}{8,000} = 0.2625$$

$$W_B = \frac{2,000 + 1,500}{8,000} = 0.4375$$

$$W_C = \frac{1,500 + 900}{8,000} = 0.30$$

(II) Calculation of W_B & W_C if W_A = 0.5

$Y = a + bx$

$0.4 = a + 0.30b$ .....(i)

$0.5 = a + 0.20b$ .....(ii)

$-0.1 = 0.1b$

$b = \frac{-0.1}{0.1} = -1$ Slope

Put Slope in equation (i)

$0.4 = a + 0.30 \times -1$

$0.4 = a - 0.30$

$a = 0.70$

Equation

$Y = 0.70 + -1x$

If $W_A = 0.5$

W_B = 0.70 (-1 × 0.5)

= 0.2

W_C = 1 – 0.5 – 0.2 = 0.3

Allocation = A = 8,000 × 0.5 = 4,000

B = 8,000 × 0.2 = 1,600

C = 8,000 × 0.3 = 2,400

Question – 12

An Investor is proposing to invest ₹ 10,000/- in two Portfolios A and B in the ratio of 3:2. The Portfolios have three securities each with following weights:

	Wx	Wy	Wz
Portfolio A	0.30	0.25	0.45
Portfolio B	0.20	0.45	0.35

You are required to

(i) Calculate the weight of each stock.

(ii) Calculate the amount allocated to B and C if half of the funds are allocated to security X.

Solution:

1. Calculation of Weights

10,000 in 3:2	X	Y	Z
(6,000) Portfolio A	1,800	1,500	2,700
(4,000) Portfolio B	800	1,800	1,400
	2,600	3,300	4,100

Wx = 2600/10000 = 0.26

Wy = 3,300/10,000 = 0.33

Wz = 4,100/10,000 = 0.41

2. Calculation of Wy & Wz when Wx = 0.5 :

Using Critical Line:

Y = a + bx

0.25 = a + 0.30 .....(i)

0.45 = a + 0.20 .....(ii)

-0.20 = 0.1b

$$b = \frac{-0.20}{0.1} = -2$$

Putting b value in equation (i)

0.25 = a + (-2 × 0.30)

a = 0.85

Equation:

y = 0.85 - 2x

Wx = 0.85 (– 2 × 0.5)

Wx = - 0.15

Wz = 1 – (0.5 - 0.15)

= 0.65

Investment:

X = 10,000 × 0.5 = 5,000

Y = 10,000 × (-0.15) = (1,500) Short sale.

Z = 10,000 × 0.65 = 6,500

PART II: CAPITAL ASSET PRICING MODEL

CALCULATION OF BETA

Question – 13

The distribution of return of security 'F' and the market portfolio 'P' is given below:

Probability	Return %	
	F	P
0.30	30	-10
0.40	20	20
0.30	0	30

You are required to calculate the expected return of security 'F' and the market portfolio 'P', the covariance between the market portfolio and security and beta for the security.

(SM TYK – 09)

Solution:

Calculation of Cov_{xy}

p	x	p(x)	$(x - \bar{x})$	$(x - \bar{x})^2$ p	y	P(y)	$(y - \bar{y})$	$(y - \bar{y})^2$ p	$(x - \bar{x})$ $(y - \bar{y})$ p
0.30	30	9	13	50.7	-10	-3	-24	172.80	-93.60
0.40	20	8	3	3.60	20	8	6	14.40	7.20
0.30	0	0	-17	86.70	30	9	16	76.80	-81.60
	$\bar{x}$ =	17%		141(%)²	$\bar{y}$ =	14%		264(%)²	-168(%)²

$$x = \sqrt{141(\%)^2} = 11.87\%$$

$$\sigma_y = \sqrt{264(\%)^2} = 16.25\%$$

$$\beta = \frac{Cov_{xm}}{\sigma_m^2} = \frac{-168(\%)^2}{264(\%)^2} = -0.636$$

Question – 14

Given below is information of market rates of Returns and Data from two Companies A and B:

	Year 2007	Year 2008	Year 2009
Market (%)	12.0	11.0	9.0
Company A (%)	13.0	11.5	9.8
Company B (%)	11.0	10.5	9.5

You are required to determine the beta coefficients of the shares of Company A and Company B.

(SM TYK – 10)

Solution:

Beta of Portfolio in case of A & Market

Year	x	$(x - \bar{x})$	$(x - \bar{x})^2$	y	$(y - \bar{y})$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
1	13	1.57	2.46	12	1.33	1.77	2.09
2	11.5	0.07	0.0049	11	0.33	0.11	0.02
3	9.8	-1.63	2.66	9	-1.67	2.79	2.72
	34.3		**5.12**	**32**		**4.67**	**4.83**

$\bar{x}$ = 11.43%

$\sigma_x = \sqrt{\frac{5.12}{3}}$ = 1.31%

$\bar{y}$ = 10.67%

$\sigma_y = \sqrt{\frac{4.67}{3}}$ = 1.25%

$Cov_{xy} = \frac{4.83}{3}$ = 1.61(%)^2

$B_P = \frac{1.31}{1.25} \times 0.983$ = 1.03

$r_{xm} = \frac{1.61}{1.31 \times 1.25}$ = 0.983

Beta of Portfolio in case of B & Market

Year	x	$(x - \bar{x})$	$(x - \bar{x})^2$	y	$(y - \bar{y})$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
1	11	0.67	0.45	12	1.33	1.77	0.89
2	10.5	0.17	0.03	11	0.33	0.11	0.06
3	9.5	-0.83	0.69	9	-1.67	2.79	1.39
	31		**1.17**	**32**		**4.67**	**2.34**

$\bar{x}$ = 10.33%

$\sigma_x = \sqrt{\frac{1.17}{3}}$ = 0.62%

$\bar{y}$ = 10.67%

$\sigma_y = \sqrt{\frac{4.67}{3}}$ = 1.25%

$Cov_{xy} = \frac{2.34}{3}$ = 0.78(%)^2

$r_{xy} = \frac{0.78}{0.62 \times 1.25}$ = 1.006

$B_p = \frac{0.62}{0.125} \times 1.006$ = 0.50

Question – 15

The rates of return on the security of Company X and market portfolio for 10 periods are given below:

Period	Return of Security X (%)	Return on market portfolio (%)
1	20	22
2	22	20
3	25	18
4	21	16
5	18	20
6	-5	8
7	17	-6

8	19	5
9	-7	6
10	20	11

(i) What is the beta of Security X?

(ii) What is the characteristic line for Security X?

(SM TYK – 12)

Solution:

Year	x	$(x - \bar{x})$	$(x - \bar{x})^2$	y	$(y - \bar{y})$	$(Y - \bar{Y})^2$	$(x - \bar{x})(y - \bar{y})$
1	20	5	25	22	10	100	50
2	22	7	49	20	8	64	56
3	25	10	100	18	6	36	60
4	21	6	36	16	4	16	24
5	18	3	9	20	8	64	24
6	-5	-20	400	8	-4	16	80
7	17	2	4	-6	-18	324	-36
8	19	4	16	5	-7	49	-28
9	-7	-22	484	6	-6	36	132
10	20	5	25	11	-1	1	- 5
x =	15%	$\sum(x - \bar{x})^2$ =	1148 $(\%)^2$	y =120%		706$(\%)^2$	357$(\%)^2$

$\sigma_x = \frac{\sqrt{1148}}{10} = 10.71\%$

$\sigma_y = \frac{\sqrt{706}}{10} = 8.402\%$

$Cov_{xy} = \frac{357\%}{10} = 35.7(\%)^2$

(1) Beta $= \frac{COV_{xm}}{\sigma_m^2} = \frac{35.7}{(8.402)^2} = 0.505$

Or,

Beta $= \frac{\sigma_x}{\sigma_m} \times r_{xm}$

$$r_{xm} = \frac{COV_{xm}}{\sigma_x \times \sigma_y} = \frac{35.7}{10.71 \times 8.402} = 0.397$$

$$= \frac{10.71}{8.402} \times 0.397 = 0.505$$

Characteristic Line (CL): Characteristic Line is used to calculate Beta (β) & it starts from Alpha (α).

Equation of Characteristic Line

$\bar{y} = a + b\bar{x}$

$\bar{y} = \alpha + bR_m$

(2) Characteristic Line

$\bar{y} = \alpha + b\bar{x}$ $\bar{x}$ = Market

$15 = \alpha + 0.505 \times 12$ $\bar{y}$ = Stock

$15 = \alpha + 6.06$

$\alpha = 8.94$

Equation of Characteristic Line

$\bar{y} = 8.94 + 0.505x$

Question – 16

The following information is available in respect of Security X

Equilibrium Return	15%
Market Return	15%
7% Treasury Bond Trading at	$140
Covariance of Market Return and Security Return	225%
Coefficient of Correlation	0.75

You are required to determine the Standard Deviation of Market Return and Security Return.

(SM TYK – 19)

Solution:

First we shall compute the β of Security X

$$\text{Risk Free Rate} = \frac{\text{Coupon Payment}}{\text{Current Market Price}} = \frac{7}{140} = 5\%$$

Assuming equilibrium return to be equal to CAPM return then:

$15\% = R_f + \beta_X(R_m - R_f)$

$15\% = 5\% + \beta_X(15\% - 5\%)$

$\beta_X = 1$

or it can also be computed as follows:

$$\frac{R_m}{R_s} = \frac{15\%}{15\%} = 1$$

(i) Standard Deviation of Market Return

$$\beta_x = \frac{Cov_{X,\,M}}{\sigma_m^2} = \frac{225\%}{\sigma_m^2} = 1$$

$\sigma_m^2 = 225$

$\sigma_m = \sqrt{225} = 15\%$

(ii) Standard Deviation of Security Return

$$\beta_X = \frac{\sigma_x}{\sigma_m} \times \sigma_{xm}$$

$$= \frac{\sigma_x}{15} \times 0.75$$

$= 1$

$$\sigma_X = \frac{15}{0.75}$$

$= 20\%$

PORTFOLIO BETA & BETA MANAGEMENT

Question – 17

A Portfolio Manager (PM) has the following four stocks in his portfolio:

Security	No. of Shares	Market Price per Share (₹)	β
VSL	10,000	50	0.9
CSL	5,000	20	1.0
SML	8,000	25	1.5
APL	2,000	200	1.2

Compute the following:

(i) Portfolio beta.

(ii) If the PM seeks to reduce the beta to 0.8, how much risk free investment should he bring in?

(iii) If the PM seeks to increase the beta to 1.2, how much risk free investment should he bring in?

(SM TYK – 36)

Solution:

(I) Portfolio Beta

Security	Share	Market Price	Amount	Weight	Beta	Weight × Beta
VSL	10,000	50	5,00,000	0.416	0.9	0.375
CSL	5,000	20	1,00,000	0.0833	1.0	0.083
SML	8,000	25	2,00,000	0.166	1.5	0.250
APL	2,000	200	4,00,000	0.333	1.2	0.400
			12,00,000			1.108

(II) B_T = 0.8

$$W_A = \frac{B_T}{B_P} = \frac{0.8}{1.108} = 0.722$$

$$\text{Portfolio Amount} = \frac{12,00,000}{0.722} = 16,62,050$$

$$R_f \text{ Amount} = 16,62,050 - 12,00,000 = 4,62,050$$

Revised Portfolio Beta

Stock	Amount	Weight	Beta	Weight × Beta
VSL	5,00,000	0.301	0.9	0.271

CSL	1,00,000	0.060	1.0	0.060
SML	2,00,000	0.120	1.5	0.180
APL	4,00,000	0.241	1.2	0.289
Risk Free Asset	4,62,050	0.278	0	0
	16,62,050			0.800

(III) B_T = 1.2

$$W_A = \frac{B_T}{B_P} = \frac{1.20}{1.108} = 1.083$$

$$\text{Portfolio Amount} = \frac{12,00,000}{1.083} = 11,08,030$$

R_f Amount = 11,08,030 – 12,00,000 = -91,967

Borrow at R_f = ₹ 91,967

Revised Portfolio Beta

Stock	Amount	Weight	Beta	W × B
VSL	5,00,000	0.451	0.9	0.406
CSL	1,00,000	0.090	1	0.909
SML	2,00,000	0.181	1.5	0.272
APL	4,00,000	0.361	1.2	0.433
Risk Free Asset	-91,967	-0.083	0	0
	11,08,030			1.20

Question – 18

Mr. Tempest has the following portfolio of four shares:

Name	Beta	Investment Lac.
Oxy Rin Ltd.	0.45	0.80
Boxed Ltd.	0.35	1.50
Square Ltd.	1.15	2.25
Ellipse Ltd.	1.85	4.50

The risk free rate of return is 7% and the market rate of return is 14%.

Required.

(i) Determine the Portfolio Return.

(ii) Calculate the Portfolio Beta.

(SM TYK – 16, RTP November – 2018)

Solution:

Portfolio Beta

$$\beta_P = \frac{(0.45 \times 0.80) + (0.35 \times 1.50) + (1.15 \times 2.25) + (1.85 \times 4.50)}{9.05}$$

$$= 1.3035$$

(i) Portfolio Return

$$E_{RP} = R_f + (R_m - R_f)\, B_p$$

$$= 7 + (14 - 7)\ 1.3035$$

$$= 16.1245\%$$

(ii) Portfolio Beta

$$\beta_P = 1.3035$$

Question – 19

Mr. Fed Up wants to invest an amount of ₹ 520 lakhs and had approached his Portfolio Manager. The Portfolio Manager had advised Mr. Fed Up to invest in the following manner:

Security	Moderate	Better	Good	Very Good	Best
Amount (in ₹ Lakhs)	60	80	100	120	160
Beta	0.5	1.00	0.80	1.20	1.50

You are required to advise Mr. Fed Up in regard to the following, using Capital Asset Pricing Methodology:

(i) Expected return on the portfolio, if the Government Securities are at 8% and the NIFTY is yielding 10%.

(ii) Advisability of replacing Security 'Better' with NIFTY.

(SM TYK – 25)

Solution:

(1) Computation of Expected Return from Portfolio

Stock	Amount	Weight	Beta	Weight × Beta
Moderate	60	0.115	0.5	0.058
Better	80	0.154	1.00	0.154
Good	100	0.192	0.80	0.154
Very Good	120	0.231	1.20	0.277
Best	160	0.308	1.50	0.462
	520			1.105

$ER_P = R_f + (R_m - R_f) B_p$

$= 8 + (10 - 8)\ 1.105$

$= 10.21\%$

(2) As computed above the expected return from Better is 10% same as from Nifty, hence there will be no difference even if the replacement of security is made. The main logic behind this neutrality is that the beta of security 'Better' is 1 which clearly indicates that this security shall yield same return as market return.

Question – 20

M/s. Siri Ltd. Has a surplus amount of ₹ 3 crores to invest and has shortlisted the following equity shares:

Company	Beta
S Ltd.	1.6
K Ltd.	1
P Ltd.	-0.3
D Ltd.	2
C Ltd.	0.6

Required:

(i) If M/s. Siri Ltd. invests an equal amount in all securities, what is the beta of the portfolio?

(ii) If M/s. Siri Ltd. invests 15% of its investment in S Ltd., 15% in P Ltd., 10% in C Ltd. and the balance in equal amount in the other two securities, what is the beta of the portfolio?